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[ > # Map0003P - 2012/1
[ > # script para obtencao das principais Formulas de Quadratura
  Newtoniana
[ > # contexto: queremos uma formula para
[ > # int(phi,x=a..b) onde phi(x) eh a interpoladora
[ > # Lagrangeana de f(x) em x0 < x1 < ... < xn
[ > # onde x0 = a , xn = b, xi = x0 + i*h
[ > # usaremos Formula de Newton via diferencas finitas ascendentes
  para a Lagrangeana
[ > #
[ > # int(phi,x=a..b)=int(phi(s)*dx/ds*ds, s=0..n)
[ > # x = x0 + s*h ==> dx/ds = h
[ > #
[ > # n=1: pontos {(x0,y0),(x1,y1)}
[ > f01 := y1-y0; # temos apenas 1 diferenca
      
$$f01 := y1 - y0$$

[ > phi1:=y0+f01*s;
      
$$\phi1 := y0 + (y1 - y0)s$$

[ > s1:=(b-a)*integrate(phi1,s=0..1);
      
$$s1 := (b - a) \left( \frac{y0}{2} + \frac{y1}{2} \right)$$

[ > # esta eh a Regra do Trapezio
[ > #
[ > # n=2: pontos {(x0,y0),(x1,y1),(x2,y2)}
[ > f12:=y2-y1; # temos mais esta dif ascend
      
$$f12 := y2 - y1$$

[ > f012:=f12-f01; # dif de segunda ordem
      
$$f012 := y2 - 2y1 + y0$$

[ > phi2:= y0 + f01*s + f012*s*(s-1)/2;
      
$$\phi2 := y0 + (y1 - y0)s + \frac{(y2 - 2y1 + y0)s(s-1)}{2}$$

[ > s2:=(b-a)/2*integrate(phi2,s=0..2);
      
$$s2 := \frac{(b-a) \left( \frac{y2}{3} + \frac{4y1}{3} + \frac{y0}{3} \right)}{2}$$

[ > # esta ultima eh a Regra de Simpson
[ > #
[ > # n=3: pontos {(x0,y0),..., (x3,y3)}
[ > f23:=y3-y2; f123:=f23-f12;
      
$$f23 := y3 - y2$$

      
$$f123 := y3 - 2y2 + y1$$

[ > f0123:=f123-f012;
      
$$f0123 := y3 - 3y2 + 3y1 - y0$$

[ > phi3:=y0+f01*s+f012*s*(s-1)/2+f0123*s*(s-1)*(s-2)/6;
      
$$\phi3 := y0 + (y1 - y0)s + \frac{(y2 - 2y1 + y0)s(s-1)}{2} + \frac{(y3 - 3y2 + 3y1 - y0)s(s-1)(s-2)}{6}$$

[ > s3:=(b-a)/3*integrate(phi3,s=0..3);

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$$s3 := \frac{(b-a) \left( \frac{3y3}{8} + \frac{9y2}{8} + \frac{9y1}{8} + \frac{3y0}{8} \right)}{3}$$

[ > # esta ultima eh a regra 3/8 de Simpson

[ > #

[ > # n=4 pontos  $\{(x0, y0), \dots, (x4, y4)\}$

[ > f34:=y4-y3; f234:=f34-f23; f1234:=f234-f123;

$$f34 := y4 - y3$$

$$f234 := y4 - 2y3 + y2$$

$$f1234 := y4 - 3y3 + 3y2 - y1$$

[ > f01234:=f1234-f0123;

$$f01234 := y4 - 4y3 + 6y2 - 4y1 + y0$$

[ > phi4:=phi3 + f01234\*s\*(s-1)\*(s-2)\*(s-3)/24;

$$\phi4 := y0 + (y1 - y0)s + \frac{(y2 - 2y1 + y0)s(s-1)}{2} + \frac{(y3 - 3y2 + 3y1 - y0)s(s-1)(s-2)}{6}$$

$$+ \frac{(y4 - 4y3 + 6y2 - 4y1 + y0)s(s-1)(s-2)(s-3)}{24}$$

[ > s4:=(b-a)/4\*integrate(phi4,s=0..4);

$$s4 := \frac{(b-a) \left( \frac{14y4}{45} + \frac{64y3}{45} + \frac{8y2}{15} + \frac{64y1}{45} + \frac{14y0}{45} \right)}{4}$$

[ > # esta ultima eh a regra de Boole

[ > # reveja as formulas e os correspondentes

[ > # erros de truncamento em

[ > # [http://en.wikipedia.org/wiki/Newton-Cotes\\_formulas](http://en.wikipedia.org/wiki/Newton-Cotes_formulas)

[ > # JBC, 13/3/2012