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Algebraic foliations without algebraic solutions. (English summary)

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J. P. Jouanolou [*Équations de Pfaff algébriques*, Lecture Notes in Math., 708, Springer, Berlin, 1979; MR0537038 (81k:14008)] gave an example of a holomorphic singular foliation $\mathcal{F}$ in $\mathbf{C}\mathrm{P}_2$ without algebraic solutions having singularities of Poincaré type. Here an algebraic solution of $\mathcal{F}$ means an irreducible algebraic curve $S \subset \mathbf{C}\mathrm{P}_2$ such that $S - \mathrm{Sing}(\mathcal{F})$ is a leaf of $\mathcal{F}$. The author deals with the following two natural questions: (1) Do there exist foliations without algebraic solutions on any algebraic, compact, complex (nonsingular) surface $V_2 \subset \mathbf{C}\mathrm{P}_n$? (2) Do there exist foliations in $\mathbf{C}\mathrm{P}_2$ without algebraic solutions but admitting degenerate singularities? Concerning these questions he proves the following two results: (A) Let $V_2 \subset \mathbf{C}\mathrm{P}_n$ be an algebraic, compact, complex (nonsingular) surface. There exist foliations in V_2 without algebraic solutions, having singular set composed of singularities of Poincaré type and (possibly) singularities with local holomorphic first integral. (B) Given $\{p_1, \cdots, p_n\}$, an arbitrary finite set of points of $\mathbf{C}\mathrm{P}_2$, there exists a foliation $\mathcal{F}_0$ having dicritical singularities at $p_1, \cdots, p_n$ and such that $\mathcal{F}_0$ can be approximated by foliations $\mathcal{F}_t$, $t \in (\mathbf{C}, 0)$, without algebraic solutions, having dicritical singularities at $p_1, \cdots, p_n$.

Reviewed by *Joan Girbau*