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Kodaira dimension of holomorphic singular foliations. (English summary)

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The main goal of this article is to define and study the notion of Kodaira dimension for holomorphic (singular) foliations on complex projective surfaces. Let $\mathcal{F}$ be such a foliation, defined on a smooth projective surface X. The Kodaira dimension of $\mathcal{F}$ is defined as follows: first, take a reduced model $\mathcal{F}_{\rm red}$ of $\mathcal{F}$, which means that each singularity is reduced in the sense of Seidenberg (this is always possible by blowing up X) ; then define $\text{kod}(\mathcal{F})$ to be the Kodaira dimension of the cotangent bundle of $\mathcal{F}_{\rm red}$. This dimension $\text{kod}(\mathcal{F})$ is invariant under birational transformations of reduced foliations (Theorem 3.1.1). If there exists a Zariski-dense entire mapping $f\colon \mathbf{C} \to X$ which is tangent to $\mathcal{F}$, then $\text{kod}(\mathcal{F}) \leq 1$ (Remark 3.3.4). If some leaf of $\mathcal{F}$ is a generic fiber of an elliptic fibration, or if $\mathcal{F}$ is generically transverse to a fibration, the same inequality holds.

Conversely, if $\text{kod}(\mathcal{F})$ equals 1, then $\mathcal{F}$ either is generically transverse to a fibration or coincides with an elliptic fibration (Theorem 3.3.1). Applying Miyaoka's semipositivity theorem, the author also proves that if $\text{kod}(\mathcal{F}) = -\infty$ then $\mathcal{F}$ is birationally equivalent to a rational fibration, or $\mathcal{F}$ is not deformable.

The last section defines a notion of genus $g(\mathcal{F})$ for foliations of surfaces and studies this notion.

This is a nice and well-written paper. Interesting examples are given to illustrate the different theorems.

Reviewed by *Serge Cantat*