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Calvo-Andrade, Omegar; **Mendes, Luís Gustavo** (BR-UFRS); **Pan, Ivan** (BR-UFRS)
Foliations with radial Kupka set and pencils of Calabi-Yau hypersurfaces. (English summary)
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This work considers codimension one holomorphic foliations $\mathcal{F}$ on complex projective spaces $\mathbb{CP}(n)$, $n \geq 3$, whose singular set admits a compact Kupka component, say $K(\mathcal{F})$. In this case, $K(\mathcal{F})$ is smooth and the other components of the singular set of $\mathcal{F}$ do not intersect it. We recall that the degree of $\mathcal{F}$ is the number of tangencies of its leaves with a generic projective line $\mathbb{CP}(1) \subset \mathbb{CP}(n)$. By cutting such a foliation by a generic projective plane $\mathbb{CP}(2) \subset \mathbb{CP}(n)$, we obtain a one-dimensional holomorphic foliation $\mathcal{F}'$, on this $\mathbb{CP}(2)$, and the Baum-Bott indices of a singularity of $\mathcal{F}'$ are called sectional Baum-Bott indices of $\mathcal{F}$. The main result of the paper is as follows: suppose (i) $\mathcal{F}$ has degree $2n$, (ii) that all the sectional Baum-Bott indices of $\mathcal{F}$, arising from the components of the singular set of $\mathcal{F}$ which are distinct from $K(\mathcal{F})$, are non-positive and (iii) that, after blowing-up $\mathbb{CP}(n)$ along $K(\mathcal{F})$ and obtaining a transformed foliation $\widehat{\mathcal{F}}$, we have that $\bigwedge^{n-1} T^*_{\widehat{\mathcal{F}}}$ is nef (meaning $c_1(\bigwedge^{n-1} T^*_{\widehat{\mathcal{F}}}) \cdot C \geq 0$ for all curves C). Then, the codimension 2 part of the singular set of $\mathcal{F}$ consists only of $K(\mathcal{F})$, the degree of $K(\mathcal{F})$ is $(n+1)^2$ and $\mathcal{F}$ is a pencil of hypersurfaces of degree $n+1$ which are smooth along the base locus $K(\mathcal{F})$.

Reviewed by *M. G. Soares*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.