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**MR2142243 (2006d:32043)** 32S65 (11F41 14G35 37F75)
**Mendes, L. G.** (BR-IMPA); **Pereira, J. V.** (BR-IMPA)
**Hilbert modular foliations on the projective plane. (English summary)**
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Let $\mathbb{D}$ be the open unit disc in the complex line $\mathbf{C}$. The group of holomorphic diffeomorphisms of the disc coincides with the group of Möbius transformations $\mathrm{PSL}(2,\mathbf{R})$. Let $d$ be a squarefree positive integer and let $\mathcal{O}_d$ be the ring of integers in $\mathbf{Q}[\sqrt{d}]$. Let $\Gamma_d$ be the discrete subgroup of $\mathrm{PSL}(2,\mathbf{R})\times\mathrm{PSL}(2,\mathbf{R})$ obtained by a twisted diagonal embedding of $\mathrm{PSL}(2,\mathcal{O}_d)$: the image of an element $M$ in $\mathrm{PSL}(2,\mathcal{O}_d)$ is $(M,\sigma(M))$ where $\sigma(\sqrt{d})=-\sqrt{d}$.

The Hilbert modular surface $S_d$ associated to this construction is the quotient of the bidisc $\mathbb{D}\times\mathbb{D}$ by the action of $\Gamma_d$. This surface is singular but there is a natural compactification $\overline{S}_d$ of it. The foliation of the bidisc by horizontal lines is invariant under the action of $\Gamma_d$ and defines an algebraic (singular) foliation on the projective surface $\overline{S}_d$.

These Hilbert modular foliations play an important role in the recent classification of holomorphic foliations of compact Kähler surfaces by Brunella, Mendes and McQuillan. The main goal of this paper is to describe explicitly the foliation in the case where $\overline{S}_d$ is the projective plane $\mathrm{P}^2(\mathbf{C})$.

The authors focus on the case $d=5$ (and some other related examples). They obtain a beautiful and complete description with explicit equations and nice pictures linked to the geometry and invariant theory of the icosahedral group.

Reviewed by *Serge Cantat*

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