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The Noether-Fano inequalities for codimension one singular holomorphic foliations. (English summary)

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Let $\mathcal{F}$ be a codimension 1 holomorphic foliation on $\mathbb{CP}^N$, and let $\chi\colon \mathbb{CP}^N \dashrightarrow \mathbb{CP}^N$ be any birational map. The main theorem of this paper gives some conditions on the singularities of $\mathcal{F}$ at the indeterminacy set of χ so that $\deg(\chi_*\mathcal{F}) \geq \deg(\mathcal{F})$. The proof is inspired by the proof of the classical Noether-Fano inequalities. It amounts to the study of a suitable $\mathbb{Q}$-divisor involving the canonical divisor of $\mathbb{CP}^N$ and the normal bundle of $\mathcal{F}$ in a resolution of the graph of χ.

Numerous examples are then treated to illustrate this result. The degree of the image of a foliation by the standard Cremona transformation in dimensions 2 and 3, and by the cubo-cubic Cremona map in dimension 3 is computed. A nice and simple criterion for proving the minimality of $\deg(\mathcal{F})$ in its birational class is given in dimension 2. It is used to prove the minimality of the degree of some modular foliations considered in [L. G. Mendes and J. V. Pereira, Comment. Math. Helv. **80** (2005), no. 2, 243–291; MR2142243 (2006d:32043)].

Reviewed by *Charles Favre*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.