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Brunella, Marco (F-DJON-T); **Gustavo Mendes, Luís** (F-DJON-T)
Bounding the degree of solutions to Pfaff equations. (English summary)
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This article presents new results concerning the following question posed by Poincaré: Given a foliation $\mathcal{F}$ on $\mathbf{C}\mathrm{P}(n)$, of degree d, and an $\mathcal{F}$-invariant hypersurface $V \subset \mathbf{C}\mathrm{P}(n)$, is it possible to bound the degree of V by a number $h(d)$ depending only on d and not on $\mathcal{F}$? It is well known that the answer to this question, in all its generality, is no; however, several positive results have been obtained [see E. Ballico, Ann. Global Anal. Geom. **14** (1996), no. 3, 257–261; MR1400289 (97d:32044); M. Brunella, Publ. Mat. **41** (1997), no. 2, 527–544; MR1485502 (98m:32052); M. M. Carnicer, Ann. of Math. (2) **140** (1994), no. 2, 289–294; MR1298714 (95k:32031); D. Cerveau and A. Lins Neto, Ann. Inst. Fourier (Grenoble) **41** (1991), no. 4, 883–903; MR1150571 (93b:32050); M. G. Soares, Invent. Math. **128** (1997), no. 3, 495–500; MR1452431 (99a:32043)]. In this work the authors consider the more general context of Pfaff equations, which are defined as follows: Given a complex manifold X and a holomorphic line bundle N on X, a Pfaff equation of codimension p, $1 \leq p \leq \dim X - 1$, is a nontrivial global section σ of $\Omega^p_X \otimes N$, where Ω^p_X denotes the sheaf of holomorphic p-forms on X. With this at hand, a hypersurface V is said to be invariant by the Pfaff equation σ if $i_V \sigma \equiv 0$. The main result is then the following: Let X be a complex projective manifold whose Picard group is $\mathbf{Z}$, let $\sigma \in H^0(\Omega^p_X \otimes N)$ be a Pfaff equation on X, and V be a normal crossings hypersurface invariant by σ. Then $\mathrm{degree}(V) \leq \mathrm{degree}(N)$; the inequality is strict if V is smooth. Moreover, if $\mathrm{degree}(V) = \mathrm{degree}(N)$, then σ is given by a global closed logarithmic p-form on X with poles along V.

Reviewed by *M. G. Soares*

[References]

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

1. E. Ballico, Meromorphic singular foliations on complex projective surfaces, *Ann. Global Anal. Geom.* **14(3)** (1996), 257–261. MR1400289 (97d:32044)
2. F. A. Bogomolov, Unstable vector bundles and curves on surfaces, in: "*Proceedings of the International Congress of Mathematicians*" (Helsinki, 1978), Acad. Sci. Fennica, 1980, pp. 517–524. MR0562649 (81k:14013)
3. M. Brunella, Some remarks on indices of holomorphic vector fields, *Publ. Mat.* **41(2)** (1997), 527–544. MR1485502 (98m:32052)
4. M. M. Carnicer, The Poincaré problem in the nondicritical case, *Ann. of Math. (2)* **140(2)** (1994), 289–294. MR1298714 (95k:32031)
5. D. Cerveau and A. Lins Neto, Holomorphic foliations in $\mathbb{C}P(2)$ having an invariant algebraic curve, *Ann. Inst. Fourier (Grenoble)* **41(4)** (1991), 883–903. MR1150571 (93b:32050)
6. P. Deligne, Théorie de Hodge. II, *Inst. Hautes Études Sci. Publ. Math.* **40** (1971), 5–57. MR0498551 (58 #16653a)
7. J.-P. Demailly, L^2 vanishing theorems for positive line bundles and adjunction theory, in "*Transcendental methods in algebraic geometry*" (Cetraro, 1994), Lecture Notes in Math. **1646**, Springer, Berlin, 1996, pp. 1–97. MR1603616 (99k:32051)
8. S. Iitaka, On D-dimensions of algebraic varieties, *J. Math. Soc. Japan* **23** (1971), 356–373. MR0285531 (44 #2749)
9. A. Lins Neto, Some examples for poincaré problem and a question of Brunella, Preprint IMPA, 1999.
10. J. Noguchi, A short analytic proof of closedness of logarithmic forms, *Kodai Math. J.* **18(2)** (1995), 295–299. MR1346909 (96g:32022)
11. P. Painlevé, "*Oeuvres de Paul Painlevé. Tome I*", Éditions du Centre National de la Recherche Scientifique, Paris, 1973. MR0532682 (58 #27154a)
12. H. Poincaré, Sur l'intégration algébrique des équations différentielles du ler ordre et du ler degré, *Rend. Circ. Mat. Palermo* **5** (1891), 161–191.
13. M. Reid, Bogomolov's theorem $c_1^2 \leq 4c_2$, in: "*Proceedings of the International Symposium on Algebraic Geometry*" (Kyoto Univ., Kyoto, 1977), Kinokuniya Book Store, 1978, pp. 623–642. MR0578877 (82b:14014)
14. K. Saito, Theory of logarithmic differential forms and logarithmic vector fields, *J. Fac. Sci. Univ. Tokyo Sect. IA Math.* **27(2)** (1980), 265–291. MR0586450 (83h:32023)
15. M. G. Soares, The Poincaré problem for hypersurfaces invariant by one-dimensional foliations, *Invent. Math.* **128(3)** (1997), 495–500. MR1452431 (99a:32043)